ITERATION

PROCESS TO FIND SOLUTIONS OF EQUATIONS THOSE CANNOT BE SOLVED OTHERWISE.

$$x^3 - e^{3x} + \tan x = 5$$

9 The equation $x^3 - 2x - 2 = 0$ has one real root.

(i) Show by calculation that this root lies between $x = 1$ and $x = 2$. [2]

(ii) Prove that, if a sequence of values given by the iterative formula

$$x_{n+1} = \frac{2x_n^3 + 2}{3x_n^2 - 2}$$

converges, then it converges to this root. [2]

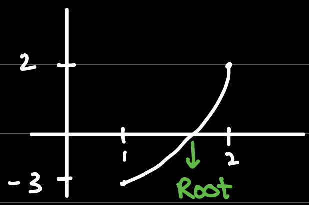
(iii) Use this iterative formula to calculate the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

(i) Bring everything to one side and replace "Zero" with y.

$$x^3 - 2x - 2 = 0$$

$$x^3 - 2x - 2 = y$$

$$y = x^3 - 2x - 2$$



$x = 1$

$y = (1)^3 - 2(1) - 2$

$y = -3$

$x = 2$

$y = (2)^3 - 2(2) - 2$

$y = 2$

If the sign of y changes, that means root lies between $x = 1$ & $x = 2$.

(ii) Prove that, if a sequence of values given by the iterative formula

$$x_{n+1} = \frac{2x_n^3 + 2}{3x_n^2 - 2}$$

converges, then it converges to this root. [2]

Process : remove all subscripts (n/n+1) and re arrange formula to make equation in first part.

$$x = \frac{2x^3+2}{3x^2-2}$$

$$3x^3 - 2x = 2x^3 + 2$$

$$3x^3 - 2x^3 - 2x - 2 = 0$$

$$x^3 - 2x - 2 = 0 \quad \text{(shown)}$$

$$x_{n+1} = \frac{2x_n^3 + 2}{3x_n^2 - 2}$$

(iii) Use this iterative formula to calculate the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

x_1 = average of values in part ii) if x_1 is not given

$$x_1 = \frac{1+2}{2} = 1.5$$

$$x_2 = \frac{2(1.5)^3 + 2}{3(1.5)^2 - 2} = 1.8421$$

Calculator: (Radians)

1.5 $\boxed{=}$

$(2\,\text{Ans}^3+2) \div (3\,\text{Ans}^2-2)$

=

=

=

$x_3 = 1.7728$

$x_4 = 1.7693$

$x_5 = 1.7693$ Root : $\boxed{x = 1.77}$